

Research Article

T-S Fuzzy-Based Optimal Control for Minimally Invasive Robotic Surgery with Input Saturation

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A minimally invasive surgery robot is difficult to control when actuator saturation exists. In this paper, a Takagi-Sugeno fuzzy model-based controller is designed for a minimally invasive surgery robot with actuator saturation, which is difficult to control. The contractively invariant ellipsoid theorem is applied for the actuator saturation. The proposed scheme can be derived using the H -infinity control theorem and parallel distributed compensation. The result is rebuilt in the form of linear matrix inequalities for easier calculation by computer. Meanwhile, the uniformly ultimately bounded stable and the prescribed H -infinity control performance can be guaranteed. The proposed scheme is simulated in a Novint Falcon haptic device system.

1. Introduction

In minimally invasive robotic surgery, the work space is limited precisely. The mechanical structure and the electrical characteristics can be additional constraint boundaries for system inputs and outputs. Therefore, the controller of the surgery robot has a rigid input saturation requirement. However, if input saturation occurs, the output performance of the controlled object cannot satisfy the designed requirement, which can result in the decline of the closed-loop system response. The output overshoot cannot be suppressed well, even becoming unstable [1, 2]. This situation is prohibited in a minimally invasive robotic surgery.

Many researchers investigated the input saturation problem and provided some solutions. Buckley [3] proposed an anti-reset windup method for the integral saturation problem. The error between the controller output and object

input was used as a complementary feedback for the controlled system. Hanus et al. [4] proposed the condition technique, where the controller output continuously tracks for a new reference input and is located out of the saturation area to avoid the saturation case. The pole-placement method was applied to the antiwindup control, which assigns the poles of nonlinear system in the desired disk for stable analysis in [5]. Although the variable structure antiwindup controller showed a good performance in the integrator windup case [6], some preset parameters were needed that were difficult to determine because expert knowledge was required. Some antiwindup controllers were also based on observer [7], internal model control [8], saturation feedback control [9], and dynamic complement [10].

The T-S fuzzy theorem, which is an important part of the fuzzy control theorem, was proposed by Takagi and Sugeno in 1985 [11]. It is utilized in both system stable control and

model identification [11, 12]. Specifically, it has a good performance in nonlinear system control [12]. The T-S fuzzy model, which is the summation of the product of the T-S fuzzy local models and their corresponding membership functions, can approximate the nonlinear system under an arbitrary degree of accuracy [12]. The T-S fuzzy model is nonlinear, and its controller design is difficult. Consequently, a procedure called parallel distributed compensation (PDC) makes the T-S fuzzy controller design easier. PDC was proposed by Wang et al. in 1995 [13]. The fuzzy sets of the PDC controller are similar with the fuzzy model. Under each controller rule, a controller can be designed for the local T-S model. By summation of the product of these controllers and their corresponding membership functions, the total controller of a nonlinear system can be represented. System stability is usually considered by the Lyapunov function. Although PDC provides a design procedure for the T-S fuzzy controller, the calculation of controller gains for global stability is still difficult, particularly in the presence of many controller rules. A numerical optimization method called linear matrix inequalities (LMIs) solved this kind of problem. LMI was defined by Willems in 1971 [14]. Neestorv and Nemirovskii [15] proposed an interior point method, which can directly solve the LMI convex optimization problem. LMI was first applied to the T-S fuzzy system stabilization analysis in [16]. In the succeeding years, LMI became the focus of increasing number of researchers [17]. Except for a few special cases without analytical solution, LMIs are generally efficient [11]. Furthermore, the LMI toolbox software provides a direct shortcut to the computer solution of LMIs. Because PDC and LMIs provide a better T-S fuzzy system controller design, they are applied in the proposed controller design in this paper.

For the advantage of the T-S fuzzy theorem, it was widely used to deal with the actuator saturation problem. In [18], based on the T-S fuzzy model, a robust dissipative controller was designed for the multiple-input multiple-output (MIMO) system with saturated time-delay input and parameter uncertainty. Their results show that the closed-loop system can be stable, but the stabilization time cannot be guaranteed. For this problem, [19, 20] provided a finite-time control by optimal control and estimated the attraction domain. The combination of the T-S fuzzy model and optimal control show an animated controller design of a nonlinear system with actuator saturation [21]. For optimal control, Hu et al. [22] provided a useful control method for the saturation system with actuator saturation based on a contractively invariant ellipsoid. This study was continued in her work by BMIs [8]. Consequently, many researchers focused on actuator saturation problems, where the T-S fuzzy based controller design method was popular. The Lyapunov stability criterion-based PDC fuzzy controller was designed for the actuator saturation system [23, 24]. Many researchers contributed in completing this theorem [25, 26]. In this paper, the result in Hu et al. [22] was adopted and rebuilt into a set of LMIs. Meanwhile, a predetermined H_∞ norm was satisfied.

This paper is organized as follows. The basic T-S fuzzy theorem is presented in Section 2. The proposed solution for the actuator saturation problem is presented in Section

3. The simulation is presented in Section 4. The paper concludes in Section 5.

2. General T-S Fuzzy Model and Control

In this paper, the considered nonlinear system with disturbance $W(t)$ is

$$\dot{X}(t) = f(X(t)) + g(X(t))U(t) + W(t), \quad (1)$$

where $X(t) \in R^{n \times 1}$, $f(X(t)) \in R^{n \times 1}$, $g(X(t)) \in R^{n \times m}$, $U(t) \in R^{m \times 1}$, and $\|W(t)\| \leq W_b$ and W_b is the boundary of disturbance. In the T-S fuzzy model, the premise variables of the T-S fuzzy rules must be measurable, and they can represent some properties of the nonlinear system. Therefore, a suitable selection of these variables is very important for the accuracy and reliability of the T-S fuzzy model. When the T-S fuzzy premise variables have been defined, a corresponding local model can be provided by these variable values. The i th rule of the T-S fuzzy system for a nonlinear system is as follows:

The i th rule: If $z_1(t)$ is M_{i1} , ..., and $z_p(t)$ is M_{ip} , then

$$\dot{X}(t) = A_i X(t) + B_i U(t), \quad (2)$$

where $i = 1, 2, \dots, L$ is the rule number, $A_i \in R^{n \times n}$ and $B_i \in R^{n \times m}$ are the local model parameter matrices, z_1, z_2, \dots, z_p are the premise variables, and M_{i1}, \dots, M_{ip} are the fuzzy sets. The nonlinear system can be approximated by the overall T-S fuzzy system

$$\dot{X}(t) = \sum_{i=1}^L h_i(Z(t)) \{A_i X(t) + B_i U(t)\}, \quad (3)$$

where $Z(t) = [z_1(t) z_2(t) \dots z_p(t)]$ and $h_i(Z(t)) = \mu_i(Z(t)) / \sum_{i=1}^L \mu_i(Z(t))$, $\mu_i(Z(t)) = \prod_{j=1}^p \theta_{ij}(z_j(t))$ in which $\theta_{ij}(Z(t))$ is the grade of the membership of $z_j(t)$ in M_{ij} . Note that the membership function should satisfy the following equation:

$$\begin{aligned} h_i(Z(t)) &\geq 0, \\ \sum_{i=1}^L h_i(Z(t)) &= 1. \end{aligned} \quad (4)$$

Currently, the nonlinear system of (1) is changed to the T-S fuzzy model (3). Because the local T-S model of (2) is linear, its feedback controller can be designed easily as

$$U_j = K_j X(t), \quad (5)$$

where $K_j \in R^{m \times n}$ and $j = 1, 2, \dots, L$. By PDC and similar fuzzy rules of (2), the overall T-S fuzzy controller is designed as

$$U(t) = \sum_{j=1}^L h_j(Z(t)) K_j X(t). \quad (6)$$

3. H_∞ Robust Controller Design for Input Saturation

The traditional T-S fuzzy theorem cannot provide results in the control problem for a nonlinear system with input saturation. In this study, the proposed method can solve the input saturation problem. The controlled closed-loop system can be uniformly ultimately bounded (UUB) stable, and the prescribed H -infinity norm can be guaranteed. Furthermore, the result is shown in LMIs, which can be directly solved by programming.

Considering the saturation and uncertainty, the nonlinear system of (1) can be

$$\dot{X}(t) = f(X(t)) + g(X(t)) \cdot \text{Sat}(U(t)) + W(t), \quad (7)$$

where

$$\text{Sat}(U(t)) = \begin{cases} \text{sgn}(U(t)) \cdot b, & \|U(t)\| > b, \\ U(t), & \|U(t)\| \leq b, \end{cases} \quad (8)$$

$b > 0$, is the boundary value of the saturation function, and the other symbols are similar with (1). Following (3), the T-S fuzzy model can be

$$\dot{X}(t) = \sum_{i=1}^L h_i [A_i X(t) + B_i \text{Sat}(U(t))], \quad (9)$$

where

$$U(t) = \sum_{j=1}^L h_j(Z(t)) U_j(t). \quad (10)$$

Based on the work of Hu et al. [1], for the feedback controller, $U_j(t) = K_j X(t)$, if the ellipsoid $\varepsilon(P, 1)$ is contractively invariant, and matrixes $P > 0$, K_j , and H ($m \times n$ matrix) make the initial value of $X_0 \in \varepsilon(P, 1)$ and

$$\varepsilon(P, 1) \subset \ell(H), \quad (11)$$

where the polyhedron $\ell(H) := \{X(t) \in R^n : |H_j X(t)| \leq 1, j \in [1, m]\}$ and H_j is the j th row of H . The saturation feedback controller $\text{Sat}(U(t))$ satisfies

$$\text{Sat}(U_j(t)) = \text{Sat}(K_j X(t)) \in \text{co} \cdot \{D_r K_j X(t) + D_r^- H_j X(t), r \in [1, 2^m]\}, \quad (12)$$

where D_r is an m -by- m diagonal matrix (diagonal elements are 0 or 1) and $D_r^- = I - D_r$ with $r \in [1, 2^m]$.

If it is only applied to the T-S fuzzy local model, the global stability of the nonlinear system cannot be satisfied. For global stability, some changes are needed. By (7) and (9), the nonlinear system with disturbance is as follows:

$$\begin{aligned} \dot{X}(t) &= f(X(t)) + g(X(t)) \text{Sat}(U(t)) + W(t) \\ &= \sum_{i=1}^L h_i(Z(t)) \cdot (A_i X(t) + B_i \text{Sat}(U(t))) + f(X(t)) \\ &\quad - \sum_{i=1}^L h_i(Z(t)) A_i \cdot X(t) \\ &\quad + \left(g(X(t)) - \sum_{i=1}^L h_i(Z(t)) B_i \right) \text{Sat}(U(t)) + W(t). \end{aligned} \quad (13)$$

Equation (12) can be guaranteed if each D_r satisfies the following inequality:

$$\begin{aligned} \dot{X} &\leq \sum_{i=1}^L h_i(Z(t)) \left(A_i X(t) + B_i \sum_{j=1}^L h_j(Z(t)) (D_r K_j X(t) + D_r^- H_j X(t)) \right) \\ &\quad + \left(f(X(t)) - \sum_{i=1}^L h_i(Z(t)) A_i X(t) \right) \\ &\quad + \left(g(X(t)) - \sum_{i=1}^L h_i(Z(t)) B_i \right) \\ &\quad \cdot \left(\sum_{j=1}^L h_j(Z(t)) (D_r K_j X(t) + D_r^- H_j X(t)) \right) + W(t) \\ &= \sum_{i=1}^L \sum_{j=1}^L h_i(Z(t)) h_j(Z(t)) (A_i + B_i (D_r K_j + D_r^- H_j)) X(t) \\ &\quad + \Delta f + \Delta g + W(t), \end{aligned} \quad (14)$$

where

$$\begin{aligned} \Delta f &= \left(f(X(t)) - \sum_{i=1}^L h_i(Z(t)) A_i X(t) \right), \\ \Delta g &= \sum_{i=1}^L h_i(Z(t)) \sum_{j=1}^L h_j(Z(t)) (g(X(t)) - B_i) \\ &\quad \cdot (D_r K_j + D_r^- H_j) X(t). \end{aligned} \quad (15)$$

Some $\|\delta_{A_i}\| \leq 1$, $\|\delta_{B_i}\| \leq 1$, A_i , and B_i can be found and satisfy the following inequality:

$$\begin{aligned} \|\Delta f\| &= \left\| \sum_{i=1}^L h_i(Z(t)) \Delta A_i X(t) \right\| \leq \left\| \sum_{i=1}^L h_i(Z(t)) \delta_{A_i} A_i X(t) \right\| \\ &\leq \|A_r X(t)\|, \\ \|\Delta g\| &\leq \left\| \sum_{j=1}^L h_j(Z(t)) B_i (D_r K_j + D_r^- H_j) X(t) \right\|, \end{aligned} \quad (16)$$

where $\Delta A_i = \delta_{A_i} A_i$ and $\Delta B_i = \delta_{B_i} B_i$. H_∞ control performance is defined as

$$\frac{\int_0^{t_f} X^T(t) Q X(t) dt}{\int_0^{t_f} W^T(t) W(t) dt} < \rho, \quad (17)$$

where t_f is the terminal time, \mathbf{Q} is a positive-definite matrix, and ρ is the prescribed H_∞ norm, which is greater than 0 and less than 1. If ρ is minimized, the effect of $W(t)$ on $X(t)$ is minimized. Considering the initial condition, (17) can be changed as

$$\int_0^{t_f} X^T(t)QX(t)dt < X^T(0)PX(0) + \rho \int_0^{t_f} W^T(t)W(t)dt, \quad (18)$$

where \mathbf{P} is some symmetric positive-definite weighting matrix. Set the Lyapunov function as

$$V(t) = X^T(t)PX(t), \quad (19)$$

where $\mathbf{P} > 0$. For simplicity, (t) is omitted in the following sections. Then, the following theorem can be obtained.

Theorem 1. Under condition (11), if controller (12) is applied to a nonlinear system (7) and there exists a positive-definite matrix $\mathbf{P} > 0$, such that the following matrix inequalities

$$\begin{aligned} & PA_i + A_i^T P + PB_i(D_r K_j + D_r^- H_j) + (D_r K_j + D_r^- H_j)^T \\ & \cdot B_i^T P + A_t^T A_t + (D_r K_j + D_r^- H_j)^T B_t^T B_t (D_r K_j + D_r^- H_j) \\ & + 2PP + \rho^{-2}PP + Q_i < 0 \end{aligned} \quad (20)$$

are satisfied for each i, j , and r , then the closed-loop system is UUB, and the H_∞ control performance (18) is guaranteed as prescribed ρ^2 .

Proof 1. Using (14), the derivative of $V(t)$ is

$$\begin{aligned} \dot{V}(t) &= X^T P \dot{X} + \dot{X}^T P X = X^T P \left\{ \sum_{i=1}^l h_i [A_i X + B_i \text{Sat}(U) + W] \right\} \\ &+ \left\{ \sum_{i=1}^l h_i [A_i X + B_i \text{Sat}(U) + W] \right\}^T P X \leq X^T P \\ &\cdot \left\{ \sum_{i=1}^l \sum_{j=1}^L h_i h_j (A_i + B_i (D_r K_j + D_r^- H_j)) X + \Delta f + \Delta g + W \right\} \\ &+ \left\{ \sum_{i=1}^l \sum_{j=1}^L h_i h_j (A_i + B_i (D_r K_j + D_r^- H_j)) X + \Delta f + \Delta g + W \right\}^T \\ &\cdot P X \leq \sum_{i=1}^l \sum_{j=1}^L h_i h_j \left\{ X^T \left(P(A_i + B_i (D_r K_j + D_r^- H_j)) \right. \right. \\ &\quad \left. \left. + (A_i + B_i (D_r K_j + D_r^- H_j))^T P \right) X \right. \\ &\quad \left. + (\Delta f)^T (\Delta f) + X^T P P X + (\Delta g)^T (\Delta g) \right. \\ &\quad \left. + X^T P P X \right\} + X^T P W + W^T P X. \end{aligned} \quad (21)$$

Using (16), (21) can be

$$\begin{aligned} \dot{V} &\leq \sum_{i=1}^L \sum_{j=1}^L h_i h_j \left\{ X^T \left(P(A_i + B_i (D_r K_j + D_r^- H_j)) \right. \right. \\ &\quad \left. \left. + (A_i + B_i (D_r K_j + D_r^- H_j))^T P \right) X + (A_i X)^T (A_i X) \right. \\ &\quad \left. + X^T P P X + (B_i (D_r K_j + D_r^- H_j) X)^T \right. \\ &\quad \left. \cdot (B_i (D_r K_j + D_r^- H_j) X) + X^T P P X \right\} + X^T P W \\ &\quad + W^T P X \\ &= \sum_{i=1}^L \sum_{j=1}^L h_i h_j X^T \left\{ \left(P A_i + A_i^T P + P B_i (D_r K_j + D_r^- H_j) \right. \right. \\ &\quad \left. \left. + (D_r K_j + D_r^- H_j)^T B_i^T P + A_t^T A_t + (D_r K_j + D_r^- H_j)^T \right. \right. \\ &\quad \left. \left. \cdot B_t^T B_t (D_r K_j + D_r^- H_j) + 2PP \right) \right\} X - (\rho^{-1} P X - \rho W)^T \\ &\quad \cdot (\rho^{-1} P X - \rho W) + \rho^2 W^T W + \rho^{-2} X^T P P X \\ &\leq \sum_{i=1}^L \sum_{j=1}^L h_i h_j X^T \left\{ \left(P A_i + A_i^T P + P B_i (D_r K_j + D_r^- H_j) \right. \right. \\ &\quad \left. \left. + (D_r K_j + D_r^- H_j)^T B_i^T P + A_t^T A_t \right. \right. \\ &\quad \left. \left. + (D_r K_j + D_r^- H_j)^T B_t^T B_t \cdot (D_r K_j + D_r^- H_j) \right. \right. \\ &\quad \left. \left. + 2PP \right) \right\} X + \rho^2 W^T W + \rho^{-2} X^T P P X. \end{aligned} \quad (22)$$

Under the i th rule, if there is

$$\frac{\int_0^{t_f} X^T(t) (h_i^2 Q_i) X(t) dt}{\int_0^{t_f} W^T(t) W(t) dt} \leq \frac{\int_0^{t_f} X^T(t) Q_i X(t) dt}{\int_0^{t_f} W^T(t) W(t) dt} < \rho, \quad (23)$$

where $Q_i > 0$ and $\sum_{i=1}^L (h_i Q_i) > 0$, there is

$$\frac{\int_0^{t_f} X^T(t) \left(\sum_{i=1}^L h_i Q_i \right) X(t) dt}{\int_0^{t_f} W^T(t) W(t) dt} < \rho \quad (24)$$

for the nonlinear system. If $\sum_{i=1}^L (h_i Q_i) = Q$, (24) can be the H_∞ control performance defined in (17). Therefore, for each simulation time, the H_∞ control performance defined as (17) can be guaranteed. Then, in the total simulation, a prescribed H_∞ norm can be guaranteed. Meanwhile, (18) can be as follows:

$$\int_{t=0}^{t=t_f} \left(X^T \left(\sum_{i=1}^L h_i Q_i \right) X \right) dt < X^T(0)PX(0) + \rho \int_{t=0}^{t=t_f} (W^T W) dt. \quad (25)$$

If

$$\begin{aligned} & PA_i + A_i^T P + PB_i(D_r K_j + D_r^- H) + (D_r K_j + D_r^- H)^T B_i^T P \\ & + A_t^T A_t + (D_r K_j + D_r^- H)^T B_t^T B_t (D_r K_j + D_r^- H) \\ & + 2PP < -\rho^{-2}PP - Q_i, \end{aligned} \quad (26)$$

there is

$$\begin{aligned}
\dot{V} &< \sum_{i=1}^L \sum_{j=1}^L h_i h_j X^T \{-\rho^{-2} PP - Q_i\} X + \rho^2 W^T W + \rho^{-2} X^T P P X \\
&= X^T \left\{ -\rho^{-2} PP - \left(\sum_{i=1}^L h_i Q_i \right) \right\} X + \rho^2 W^T W \\
&\quad + \rho^{-2} X^T P P X = -X^T \left(\sum_{i=1}^L h_i Q_i \right) X + \rho^2 W^T W.
\end{aligned} \tag{27}$$

By integrating (27), the following result can be obtained:

$$V(t_f) - V(0) < \int_{t=0}^{t=t_f} \left(-X^T \left(\sum_{i=1}^L h_i Q_i \right) X + \rho^2 W^T W \right) dt. \tag{28}$$

The Lyapunov function (19) can be 0 when the system is stable. Therefore,

$$\int_{t=0}^{t=t_f} \left(X^T \left(\sum_{i=1}^L h_i Q_i \right) X \right) dt < X^T(0) P X(0) + \rho^2 \int_{t=0}^{t=t_f} (W^T W) dt. \tag{29}$$

The above equation is the H_∞ performance defined in (25), which satisfies (18). Note that the H_∞ norm is changed to ρ^2 .

End of proof.

In Theorem 1, it is difficult to find the solution for (20). Therefore, the transformation of inequality (20) into LMIs is necessary. By setting $R = P^{-1}$ and left and right multiplication of R , (20) can be

$$\begin{aligned}
&A_i R + R A_i^T + B_i (D_r K_j + D_r^- H_j) R + R (D_r K_j + D_r^- H_j)^T B_i^T \\
&\quad + R A_i^T A_i R + R (D_r K_j + D_r^- H_j)^T B_i^T B_i (D_r K_j + D_r^- H_j) R \\
&\quad + 2 + \rho^{-2} + R Q_i R < 0.
\end{aligned} \tag{30}$$

Let $\widehat{K}_i = K_i \cdot R$, $\widehat{H}_i = H_i \cdot R$, and $\widehat{Q}_i = (A_i^T A_i + Q_i)^{-1}$. By Schur complements, (30) can be changed as

$$\begin{bmatrix} A^* & (B_i (D_r \widehat{K}_j + D_r^- \widehat{H}_j))^T & R \\ B_i (D_r \widehat{K}_j + D_r^- \widehat{H}_j) & -I & 0 \\ R & 0 & -\widehat{Q}_i \end{bmatrix} < 0, \tag{31}$$

where $A^* = A_i R + R A_i^T + B_i (D_r \widehat{K}_j + D_r^- \widehat{H}_j) + (D_r \widehat{K}_j + D_r^- \widehat{H}_j)^T B_i^T + (2 + \rho^{-2}) I$, R , \widehat{K}_j , and \widehat{H}_j are unknown. Condition (11) is equal to

$$\begin{bmatrix} 1 & \widehat{H}_{ik} \\ \widehat{H}_{ik}^T & R \end{bmatrix} \geq 0, \tag{32}$$



FIGURE 1: Novint Falcon.

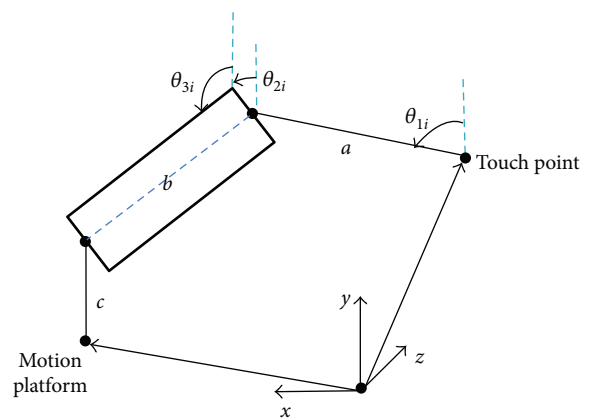


FIGURE 2: Mathematical model of the single chain.

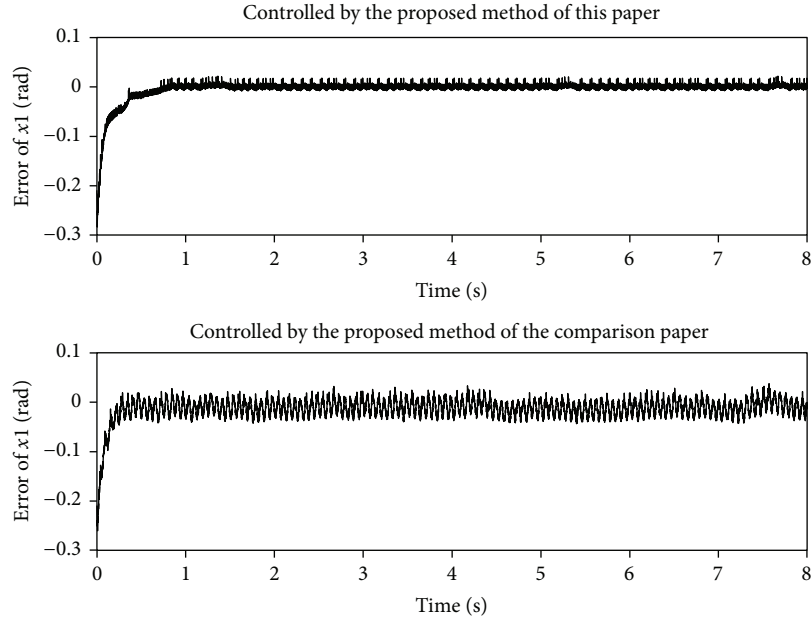
where \widehat{H}_{ik} is the k th row of \widehat{H}_i . Therefore, the following theorem is given.

Theorem 2. For nonlinear system (7) with controller (12), if there exists

$$\begin{aligned}
&\min \quad \rho^2, \\
&\text{subject to} \quad \begin{bmatrix} A^* & (B_i (D_r \widehat{K}_j + D_r^- \widehat{H}_j))^T & R \\ B_i (D_r \widehat{K}_j + D_r^- \widehat{H}_j) & -I & 0 \\ R & 0 & -\widehat{Q}_i \end{bmatrix} < 0, \\
&\quad \begin{bmatrix} 1 & \widehat{H}_{ik} \\ \widehat{H}_{ik}^T & R \end{bmatrix} \geq 0, \\
&\quad R > 0,
\end{aligned} \tag{33}$$

then the closed-loop system is UUB and the H_∞ control performance (18) is guaranteed as prescribed ρ^2 .

By this theorem, the H_∞ optimization problem can be changed to a constrained optimization problem by LMIs.

FIGURE 3: The error of x_1 .

4. Numerical Example

In minimally invasive surgery, haptic tracking is very important. Novint Falcon is a simple arm robot with force feedback ability, which benefited from parallel architecture, and can be applied in minimally invasive surgery. The Novint Falcon structure is shown in Figure 1.

The Novint Falcon consists of the base, motion platform, and 3 chains. The single chain can be abstract presented in Figure 2.

Based on [27] and ignoring some nonessentials, the i th chain dynamic equation is

$$\tau = D(\theta)\ddot{\theta} + H(\theta, \dot{\theta})\dot{\theta} + G(\theta), \quad (34)$$

where $\tau = [\tau_1, \tau_2, \tau_3]^T$ is the moment of driving force for each link; $\theta = [\theta_{1i}, \theta_{2i}, \theta_{3i}]^T$ is shown in Figure 2; $D(\theta) = I_T + (J^T)^{-1}(3m_b + m_c)J^{-1}$, where I_T is the moment of inertia and m_b and m_c are masses of links b and c ; $H(\theta, \dot{\theta}) = D_f + (J^T)^{-1}(3m_b + m_c)((J^T)^{-1})'$, where D_f denotes the damped coefficient; $G(\theta) = -M_g$ is the gravity moment;

$$\mathbf{J} = \begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{bmatrix}^{-1} \begin{bmatrix} \alpha_1 & 0 & 0 \\ 0 & \alpha_2 & 0 \\ 0 & 0 & \alpha_3 \end{bmatrix} \quad (35)$$

is the Jacobian matrix; $J_{i1} = \cos(\theta_{2i})\sin(\theta_{3i})\cos(\phi_i) - \cos(\theta_{3i})\sin(\phi_i)$, $J_{i2} = \cos(\theta_{3i})\cos(\phi_i) + \cos(\theta_{2i})\sin(\theta_{3i})\sin(\phi_i)$, $J_{i3} = \sin(\theta_{2i})\sin(\theta_{3i})$, $\alpha_i = \sin(\theta_{2i} - \theta_{1i})\sin(\theta_{3i})$, and ϕ are the angle between the chain and the coordinate frame (x, y, z) . Because the exact parameters are difficult to determine, set I_T and M_g as identity matrices and $3m_b + m_c = 1$ in this example. This is reasonable in this paper because the error between the actual

value and the required value can be distributed in the disturbance or parameter uncertainty for this system. Therefore, the matrix $D(\theta)$ is invertible. Set $\theta_{1i} = x_1(t)$, $\theta_{2i} = x_2(t)$, $\theta_{3i} = x_3(t)$, $\tau = u(t)$, and

$$\begin{aligned} \dot{x}_1(t) &= x_4(t), \\ \dot{x}_2(t) &= x_5(t), \\ \dot{x}_3(t) &= x_6(t). \end{aligned} \quad (36)$$

The θ of the other chains are set to be closed to require values, and the errors are distributed in the disturbance. Therefore, the system (34) can be as follows:

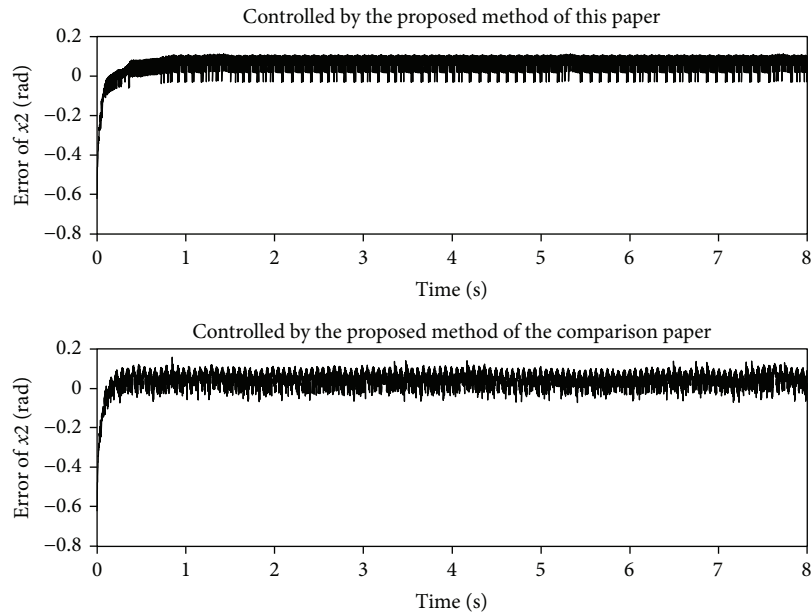
$$\begin{bmatrix} \dot{x}_4(t) \\ \dot{x}_5(t) \\ \dot{x}_6(t) \end{bmatrix} = A \begin{bmatrix} x_4(t) \\ x_5(t) \\ x_6(t) \end{bmatrix} + Bu(t) + E, \quad (37)$$

where $A = -D(\theta)^{-1}H(\theta, \dot{\theta})$, $B = D(\theta)^{-1}$, and $E = -D(\theta)^{-1}G(\theta)$. Considering actuator saturation, the corresponding i th fuzzy rule is as follows:

Rule i . If A is A_i and B is B_i , then (37) is

$$\begin{bmatrix} \dot{x}_4(t) \\ \dot{x}_5(t) \\ \dot{x}_6(t) \end{bmatrix} = A_i \begin{bmatrix} x_4(t) \\ x_5(t) \\ x_6(t) \end{bmatrix} + B_i \text{Sat}(u(t)), \quad (38)$$

where $A_1 = A(\max(\theta_{1i}), \max(\theta_{2i}), \max(\theta_{3i}))$, $A_2 = A(\min(\theta_{1i}), \max(\theta_{2i}), \max(\theta_{3i}))$, $A_3 = A(\max(\theta_{1i}), \min(\theta_{2i}), \max(\theta_{3i}))$, $A_4 = A(\max(\theta_{1i}), \max(\theta_{2i}), \min(\theta_{3i}))$, $A_5 = A(\min(\theta_{1i}), \min(\theta_{2i}), \max(\theta_{3i}))$, $A_6 = A(\min(\theta_{1i}), \max(\theta_{2i}), \min(\theta_{3i}))$, $A_7 =$

FIGURE 4: The error of x_2 .

$A(\max(\theta_{1i}), \min(\theta_{2i}), \min(\theta_{3i}))$, and $A_8 = A(\min(\theta_{1i}), \min(\theta_{2i}), \min(\theta_{3i}))$; B_i is calculated similarly, and E is treated as a disturbance. Although A_i depends on θ , the values of $\max(\theta)$ and $\min(\theta)$ do not depend on the system input. Therefore, they can be used for the fuzzy rule definition. Set $\theta_{1i} \in [0, 0.7854]$, $\theta_{2i} \in [0, 0.7854]$, and $\theta_{3i} \in [1.9199, 2.2689]$.

$\text{Sat}(u(t)) = \text{sgn}(u(t)) \cdot \min\{|u(t)|, 10\}$. The H_∞ norm is 0.3. The reference value is

$$[x_{1r} \ x_{2r} \ x_{3r} \ x_{4r} \ x_{5r} \ x_{6r}] = [0.7 \ 0.7 \ 2.2 \ 0 \ 0 \ 0]. \quad (39)$$

Using Theorem 2, the error system feedback controller gains can be calculated as follows:

$$\begin{aligned} K1 &= [-2.9 \times 10^4, -1.8 \times 10^5, 2.6 \times 10^4; 3.0 \times 10^4, 6.5 \times 10^4, -3.4 \times 10^4; -7.9 \times 10^2, 1.2 \times 10^5, 7.7 \times 10^3], \\ K2 &= [1.8 \times 10^4, 1.6 \times 10^5, -1.4 \times 10^4; -1.8 \times 10^4, -3.2 \times 10^4, 2.1 \times 10^4; -1.2 \times 10^2, -1.3 \times 10^5, -6.9 \times 10^3], \\ K3 &= [-3.8 \times 10^4, -4.7 \times 10^5, 2.2 \times 10^4; 1.4 \times 10^4, 1.1 \times 10^5, -1.2 \times 10^4; 2.4 \times 10^4, 3.6 \times 10^5, -1.1 \times 10^4], \\ K4 &= [1.9 \times 10^4, 2.9 \times 10^5, -7.6 \times 10^3; -1.7 \times 10^4, 2.2 \times 10^3, 2.2 \times 10^4; -1.4 \times 10^3, -2.9 \times 10^5, -1.4 \times 10^4], \\ K5 &= [-2.0 \times 10^4, -2.4 \times 10^5, 1.2 \times 10^4; 9.7 \times 10^3, 5.1 \times 10^4, -9.5 \times 10^3; 9.8 \times 10^3, 1.8 \times 10^5, -2.1 \times 10^3], \\ K6 &= [8.4 \times 10^3, 1.3 \times 10^5, -3.6 \times 10^3; -7.5 \times 10^3, -3.2 \times 10^3, 9.3 \times 10^3; -9.1 \times 10^2, -1.2 \times 10^5, -5.7 \times 10^3], \\ K7 &= [-1.9 \times 10^3, -3.0 \times 10^5, -1.4 \times 10^4; 4.4 \times 10^3, -1.1 \times 10^5, -1.1 \times 10^4; -2.4 \times 10^3, 4.0 \times 10^5, 2.5 \times 10^4], \\ K8 &= [9.2 \times 10^2, 6.5 \times 10^4, 2.4 \times 10^3; -2.1 \times 10^3, 1.9 \times 10^4, 3.6 \times 10^3; 1.1 \times 10^3, -8.4 \times 10^4, -6.0 \times 10^3]. \end{aligned} \quad (40)$$

To test the robustness of the controller, a disturbance

$$W(t) = \left[\sin\left(\frac{12t}{3}\right) \ 0 \ 0 \right]^T \quad (41)$$

is added in (38). For comparison, a controller designed by Theorem 2 of [18] is applied. In [18], the controller was designed for the system with input saturation and disturbance. If the time delay is ignored in [18], its results can be applied in this example. The simulation results are shown in the following figures.

From Figures 3–5, less overshooting can be observed compared with the reference paper. The proposed controller performed well with input saturation, and the H_∞ performance can be achieved.

5. Conclusion

In this paper, the T-S fuzzy control theorem and the ability of contractively invariant ellipsoid were applied for nonlinear systems with input saturation. The proposed method was

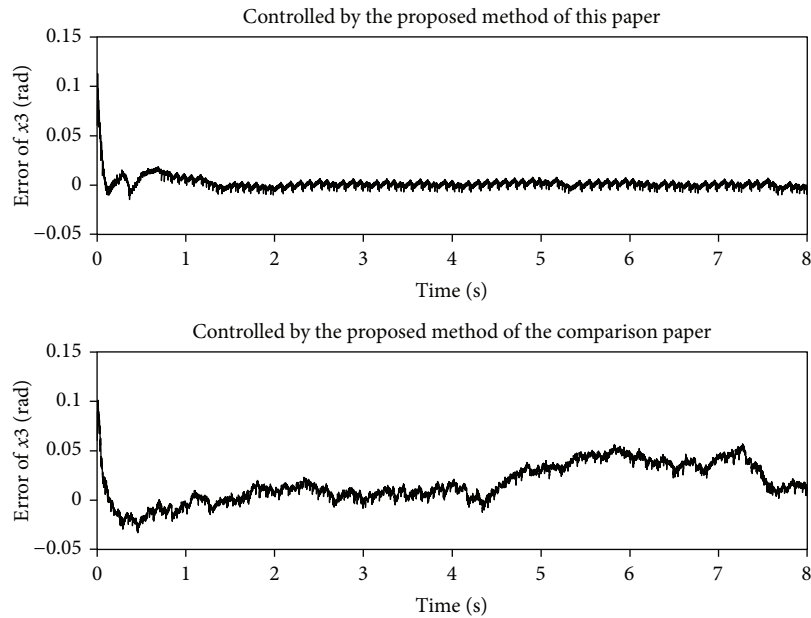


FIGURE 5: The error of x_3 .

described by LMIs, which can be calculated by a computer. The proposed theorem was tested using the Novint Falcon, and a good result was achieved. Comparing the results, the proposed method of this paper has given a better result. Meanwhile, the prescribed H -infinity norm was satisfied. In the example, only eight rules of the T-S fuzzy system were applied; there were still more than one hundred LMIs that need to be calculated. Although a feasible solution of the LMIs can be guaranteed using the proposed method, a method with less LMIs is still necessary for further study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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